HYPER VERSION

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The M2-brane soliton on the M5-brane with constant 3-form

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ABSTRACT: We obtain a BPS soliton of the effective theory of the M5-brane world-volume with constant 3-form representing M2-branes ending on the M5-brane. The dimensional reduction of this solution agrees with the known results on D-branes.

KEYWORDS: M-Theory, p-branes.

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Recently physics on the D-brane in constant NS-NS 2-form ($B_{\mu\nu}$) background is extensively studied. The worldvolume theory on the D-brane is described by either ordinary gauge theory or noncommutative gauge theory [1]. Furthermore the lift to M-theory is considered [2, 3, 4]. It is the theory on the M5-brane with constant 3-form. In this theory M2-branes ending on the M5-brane play important roles. The worldvolume theory on the D-brane with constant $B_{\mu\nu}$ has various solitons, which are interpreted as fundamental strings or D-branes ending on it. They are tilted with some angle determined by $B_{\mu\nu}$. Therefore it is natural to consider such a kind of solitons on the M5-brane i.e. M2-branes ending on the M5-brane with constant 3-form. The purpose of this paper is to obtain the BPS solution of the equations of motion representing such a configuration. The case without 3-form is considered in [5], and on special lagrangian submanifolds with constant 3-form in [6].

There are several different formulations for the effective theory of M5-brane: the covariant field equations from the superembedding approach [7], the noncovariant lagrangian [8], the covariant lagrangian with an auxiliary field [9, 10], the formulation constructed in [11], etc. The first one leads to the same field equations as those from the third one [12] and the second one is equivalent to the third one with the appropriate gauge fixing [8]. The fourth one is also shown to be equivalent to the others [11]. Therefore we can use any of these formulations for seeking solutions of the field equations. In this paper we use the first formulation.

First we will explain the notation. On the M5-brane worldvolume tangent space indices are denoted by $a, b, \ldots = 0, 1, 2, \ldots, 5$ and world indices by $m, n, \ldots = 0, 1, 2, \ldots, 5$. Indices of the target space are denoted by the same symbols with underline. The bosonic fields on the M5-brane are $X^{\underline{m}}(\sigma)$ and A_{mn}, where $X^{\underline{m}}(\sigma)$ is the embedding map from the worldvolume into the 1+10 dimensional spacetime and A_{mn} is the 2-form which satisfies the nonlinear selfduality condition explained below. The field strength of A_{mn} is always appeared in the combination $H_{mnp} = 3\partial_{[m}A_{np]} + C_{mnp}$ because of the gauge invariance for the bulk 3-form $C_{\underline{mnp}}$. (C_{mnp} is the pullback of $C_{\underline{mnp}}$.)

In this paper, we mainly consider the case where the bulk metric is flat and $C_{\underline{mnp}}$ is constant, and fermions on the M5 brane are taken to be zero since we are interested in classical solutions. The induced worldvolume metric g_{mn} and vielbein e^a_m are defined by

$$g_{mn} = \eta_{ab} e^a_m e^b_n = \partial_m X^{\underline{m}} \partial_n X^{\underline{n}} \eta_{\underline{ab}} E^{\underline{a}}_{\underline{m}} E^{\underline{b}}_{\underline{n}} \, . \tag{1}$$

We introduce the auxiliary 3-form h_{abc} satisfying the following selfdual condition.

$$h^{abc} = \frac{1}{6} \epsilon^{abcdef} h_{def} \, , \tag{2}$$

H_{mnp} is related to h_{abc} by $H_{mnp} = e^a_m (m^{-1})_b^a e^c_n e^d_p h_{bcd}$ and $m_a^b \equiv \delta_a^b - 2h_{acd}h^{bcd}$. The

κ-symmetry of the M5-brane worldvolume theory is $\delta\theta = \kappa(1+\Gamma)$ and

$$\Gamma = \frac{1}{6!\sqrt{-g}}\epsilon_{abcdef}\gamma^{abcdef} + \frac{1}{3}h_{abc}\gamma^{abc}\,, \tag{3}$$

where $\gamma_a \equiv e_a^m \partial_m X^{\underline{m}} E_{\underline{m}}^{\underline{a}} \Gamma_{\underline{a}}$ and $\Gamma_{\underline{a}}$ are the $10+1$ dimensional gamma matrices. Hereafter we take the static gauge $X^0,\ldots,X^5 = \sigma^0,\ldots,\sigma^5$ and other $X^{\underline{m}}$ are taken zero except $X \equiv X^6$. The equations of motion for X and A_{mn} are given by

$$G^{mn}\nabla_m\nabla_n X = 0\,, \tag{4}$$

$$G^{mn}\nabla_m H_{npq} = 0\,, \tag{5}$$

where $G^{mn} = e_a^m (m^2)_b^a e^{bn}$, and ∇_m is the covariant derivative with the Christoffel symbol $\Gamma^p_{mn} = \partial_m(\partial_n X^{\underline{r}} E_{\underline{r}}^{\underline{c}})\partial_q X^{\underline{s}} E_{\underline{sc}} g^{qp}$. If we take H_{mnp} constant, then in the generic case we can take all the components except h_{012} and h_{345} zero by choosing appropriate coordinate [1, 3].[1]

Let us consider the case where the M5-brane lies in the directions $X^{0,1,2,3,4,5}$ i.e. $X=0$ and $h_{012} = -h_{345} \equiv h = \text{constant}$ with other components vanishing. Then,

$$\Gamma = -\Gamma^{012345} + 2h(\Gamma^{012} - \Gamma^{345})\,. \tag{6}$$

The unbroken supersymmetry parameter ξ satisfies

$$\xi = \Gamma\xi\,. \tag{7}$$

In addition, let us consider a M2-brane ending on the M5-brane, and investigate the BPS configuration. From the case of the solitons on D-branes, we can expect that the M2-brane is tilted. Therefore we assume that it lies in the directions X^0, X^1, and $\cos\theta X^6 + \sin\theta X^2$. See figure 1.

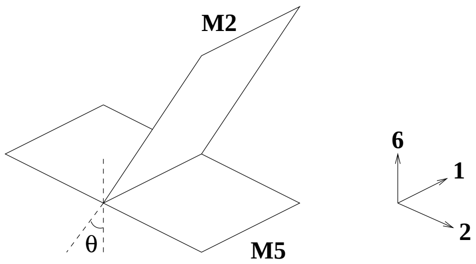

Figure 1: the M2-brane ending on the M5-brane.

Hence

$$\xi = \eta_1 \Gamma^0\Gamma^1(\cos\theta\Gamma^6 + \sin\theta\Gamma^2)\xi \equiv \eta_1\Gamma'\xi. \tag{8}$$

(Note that the κ-symmetry of the M2-brane, and therefore the unbroken supersymmetry, are independent of the 3-form.) Here $\eta_1 = \pm 1$ corresponds to the charge of the M2-brane, i.e. M2 or anti-M2. It is well known that $\theta = 0$ without the 3-form is a BPS configuration. Then

$$\Gamma\xi = \left[2\eta_1 h\sin\theta + 2\eta_1 h\cos\theta\Gamma^{26} + (-2h + \eta_1\sin\theta)\Gamma^{345} - \eta_1\cos\theta\Gamma^{23456}\right]\xi\,. \tag{9}$$

If we impose ξ the condition

$$\xi = \eta_2\Gamma^{23456}\xi\,, \tag{10}$$

[1] There is a nongeneric case where $h_{012} = -h_{512} = -h_{034} = h_{534} \equiv h$ [1, 3]. We can obtain this from the generic case by infinite boost in the 0-5 direction and taking the limit $h \to 0$.

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and $\eta_2 = \pm 1$, then

$$\Gamma\xi = \left[(2\eta_1 h\sin\theta - \eta_1\eta_2\cos\theta) + (2\eta_1\eta_2 h\cos\theta - 2h + \eta_1\sin\theta)\Gamma^{345}\right]\xi\,. \tag{11}$$

Hence there are some unbroken supersymmetries if we require

$$h = -\frac{1}{2}\frac{\eta_2\sin\theta}{\cos\theta - \eta_1\eta_2}\,. \tag{12}$$

Thus the independent conditions for ξ are eqs. (8) and (10) and eq. (7) is derived from them. This configuration preserves 1/4 supersymmetry. If we require that $h \to 0$ corresponds to $\theta \to 0$ we obtain $\eta_1\eta_2 = -1$. The nonzero component of H_{mnp} is

$$H_{012} = \frac{1}{4}\eta_1\sin\theta \tag{13}$$

$$H_{345} = \frac{1}{4}\eta_2\tan\theta\,. \tag{14}$$

Next we consider a probe M2-brane in the background representing the M5-brane with constant 3-form found in [13]:

$$\begin{aligned}
ds_{11} &= k^{1/3}f^{1/3}\Big[f^{-1}\left(-(dX^0)^2 + (dX^1)^2 + (dX^2)^2\right) + \\
&\qquad + k^{-1}\left((dX^3)^2 + (dX^4)^2 + (dX^5)^2\right) + (dX^6)^2 + \cdots + (dX^{10})^2\Big] \\
dC &= \frac{1}{4}\Big(\sin\chi df^{-1}\wedge dX^0\wedge dX^1\wedge dX^2 - \tan\chi dk^{-1}\wedge dX^3\wedge dX^4\wedge dX^5 + \\
&\qquad + \frac{1}{4!}\cos\chi\epsilon_{ijklm}\partial_m f dx^i dx^j dx^k dx^l\Big) \\
&\qquad f = 1 + \frac{R^3}{r^3}\,, \qquad k = \sin^2\chi + \cos^2\chi f \\
&\qquad i, j, \ldots = 6, 7, \ldots, 10\,,
\end{aligned} \tag{15}$$

and investigate the BPS configuration of the M2-brane to compare with the previous result. We take the following ansatz for the embedding map from the M2-brane worldvolume into the $1+10$ dimensional spacetime.

$$\begin{aligned}
X^0 &= \sigma^0 \\
X^1 &= \sigma^1 \\
\sin\theta X^2 + \cos\theta X^6 &= \sigma^2 \\
\cos\theta X^2 - \sin\theta X^6 &= 0 \\
X^{3,4,\ldots,10} &= 0\,.
\end{aligned} \tag{16}$$

Then the bosonic part of the effective action of the M2-brane is

$$\begin{aligned}
S &= -T_{M2}\int d^3\sigma\left[\sqrt{-det(\partial_m X^{\underline{m}}\partial_n X^{\underline{n}} E_{\underline{m}}^{\underline{a}} E_{\underline{na}})} - \eta_1\partial_0 X^{\underline{m}}\partial_1 X^{\underline{n}}\partial_2 X^{\underline{p}} C_{\underline{mnp}}\right] \\
&= -T_{M2}\int d^3\sigma\left[k^{1/2}f^{-1}\sqrt{\sin^2\theta + \cos^2\theta f} - \eta_1\sin\chi\sin\theta f^{-1}\right].
\end{aligned} \tag{17}$$

Here we use $C_{012} = \frac{1}{4}\sin\chi f^{-1}$ and $C_{345} = -\frac{1}{4}\tan\chi k^{-1}$. If we take $\chi = \eta_1\theta$, then

$$S = -T_{M2}\int d^3\sigma \cos^2\chi\,, \tag{18}$$

and the M2-brane has no potential term for fluctuations. This shows that this configuration is BPS. We identify the value of C_{012} and C_{345} at the infinity of the space transverse to the M5-brane with 3-form flux on the M5-brane, as was done in [14]:

$$H_{012} = \frac{1}{4}\sin\chi = \frac{1}{4}\eta_1\sin\theta\,, \qquad H_{345} = -\frac{1}{4}\tan\chi = -\frac{1}{4}\eta_1\tan\theta\,. \tag{19}$$

This value and the configuration of M2-brane (16) near the infinity of the space transverse to the M5-brane are consistent with the previous result in the flat background.

Now we solve the equations of motion of the M5-brane to obtain the BPS solution representing the above configuration in the flat background. We consider the same ansatz as in [5]:

$$\begin{aligned} h_{01a'} &= v_a \\ h_{a'b'c'} &= \epsilon_{a'b'c'd'}v^{d'}\,. \end{aligned} \tag{20}$$

with the other components of h_{abc} vanishing, and X and $v_{a'}$ are independent of σ^0 and σ^1. Here primed indices run $2,\ldots,5$. Then

$$m_a^{\ b} = \begin{pmatrix} 1+4v^2 & & \\ & 1+4v^2 & \\ & & (1-4v^2)\delta_{a'}^{b'} + 8v_{a'}v^{b'} \end{pmatrix} \tag{21}$$

$$g_{mn} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & \delta_{m'n'} + \partial_{m'}X\partial_{n'}X \end{pmatrix} \tag{22}$$

$$e_m^{\ a} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & \delta_{m'n'} + c\partial_{m'}X\partial_{n'}X \end{pmatrix} \tag{23}$$

$$e_a^{\ m} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & \delta_{m'n'} + c'\partial_{m'}X\partial_{n'}X \end{pmatrix} \tag{24}$$

$$G^{mn} = \begin{pmatrix} -(1+4v^2)^2 & & \\ & (1+4v^2)^2 & \\ & & (1-4v^2)^2 g^{m'n'} + 16v^{a'}e_{a'}^{m'}v^{b'}e_{b'}^{n'} \end{pmatrix}. \tag{25}$$

From $g_{mn} = e_m^{\ a}e_{na}$ and $g^{mn} = e_a^{\ m}e^{an}$ we can determine c and c': $c = (\partial X)^{-2}(-1\pm\sqrt{1+(\partial X)})$, $c' = (\partial X)^{-2}(-1\pm 1/\sqrt{1+(\partial X)^2})$ and $(\partial X)^2 \equiv \partial_{m'}X\partial_{n'}X\delta^{m'n'}$. We take the branch which is smoothly connected to the case $X=0$: $c=(\partial X)^{-2}(-1+\sqrt{1+(\partial X)})$, $c'=(\partial X)^{-2}(-1+1/\sqrt{1+(\partial X)^2})$. The nonvanishing components of H_{mnp} are

$$\begin{aligned} H_{01m'} &= \frac{1}{1+4v^2} e^{a'}_{m'} v_{a'} \\ H_{m'n'p'} &= \frac{\sqrt{1+(\partial X)^2}}{1-4v^2} \epsilon_{m'n'p'q'} e^{q'}_{a'} v^{a'} \,. \end{aligned} \tag{26}$$

Let us consider the BPS condition. The unbroken supersymmetry parameter ξ satisfies $\Gamma\xi=\xi$ and

$$\begin{aligned} \Gamma = {} & \frac{1}{\sqrt{1+(\partial X)^2}}(-\Gamma^{012345} + \partial_{a'} X \Gamma^{a'}\Gamma^{016}\Gamma^{2345}) + 2(v_{a'} + c'(v\partial X)\partial_{a'} X)\Gamma^{01a'} - \\ & -2\epsilon v_{a'}\Gamma^{a}\Gamma^{2345} - 2c'\left((\partial X)^2 v_{a'} - (v\partial X)\partial_{a'} X\right)\Gamma^{a'}\Gamma^{2345} + \\ & +2\frac{1}{\sqrt{1+(\partial X)^2}}(v\partial X)\Gamma^{019} + \frac{1}{\sqrt{1+(\partial X)^2}}\epsilon_{a'b'c'd'}\partial^{a'} v^{d'}\Gamma^{b'c'9} \,, \end{aligned} \tag{27}$$

where $(v\partial X) = v_{a'}\partial_{m'} X \delta^{a'm'}$. ξ should satisfy eqs. (7) and (8). We impose the condition (8) and (10) as in the discussion above and require that eq. (7) is satisfied.

$$\begin{aligned} \Gamma\xi = {} & \Bigg[\frac{1}{\sqrt{1+(\partial X)^2}}\eta_1(-\eta_2 + 2(v\partial X))\cos\theta + \\ & + \eta_1 \sin\theta\left(\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}\partial_2 X + 2c'(v\partial X)\partial_2 X + 2v_2\right)\Bigg]\xi + \\ & + \Bigg[-\frac{1}{\sqrt{1+(\partial X)^2}}\eta_1(-\eta_2 + 2(v\partial X))\sin\theta + \\ & + \eta_1\cos\theta\left(\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}\partial_2 X + 2c'(v\partial X)\partial_2 X + 2v_2\right) - \\ & - 2\eta_2\left(\frac{1}{\sqrt{1+(\partial X)^2}}v_2 - c'(v\partial X)\partial_2 X\right)\Bigg]\Gamma^{26}\xi + \\ & + \Bigg[\eta_1\cos\theta\left(\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}\partial_\alpha X + 2c'(v\partial X)v_\alpha + 2v_\alpha\right) - \\ & - 2\eta_2\left(\frac{1}{\sqrt{1+(\partial X)^2}}v_\alpha - c'(v\partial X)\partial_\alpha X\right)\Bigg]\Gamma^{\alpha 6}\xi + \\ & + \Bigg[\eta_1\sin\theta\left(\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}\partial_\alpha X + 2c'(v\partial X)v_\alpha + 2v_\alpha\right) + \\ & + 2\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}(\partial_2 X v_\alpha - v_2\partial_\alpha X)\Bigg]\Gamma^{\alpha 2}\xi + \\ & + 2\eta_2\frac{1}{\sqrt{1+(\partial X)^2}}v_\alpha\partial_\beta X\Gamma^{\alpha\beta}\xi \,. \end{aligned} \tag{28}$$

Here $\alpha, \beta, \ldots$ run $3, 4, 5$. To satisfy eq. (7), we take the coefficient of $\Gamma^{\alpha 6}$ and $\Gamma^{\alpha 2}$ zero and obtain the following BPS equations.

$$v_2 = \frac{\eta_1\eta_2}{2}\frac{\sin\theta + \cos\theta\partial_2 X + \sin\theta\frac{\partial_\alpha X\partial^\alpha X}{(\partial X)^2}}{-\eta_1\cos\theta + \eta_1\sin\theta\partial_2 X + \eta_2\sqrt{1+(\partial X)^2}} \tag{29}$$

$$v_\alpha = \frac{\eta_1\eta_2}{2}\frac{\cos\theta + \frac{1}{(\partial X)^2}\left(1 - \frac{1}{\sqrt{1+(\partial X)^2}}\right)\sin\theta\partial_2 X}{-\eta_1\cos\theta + \eta_1\sin\theta\partial_2 X + \eta_2\sqrt{1+(\partial X)^2}}\partial_\alpha X\,. \tag{30}$$

then by straightforward calculation

$$\Gamma\xi = \xi\,, \tag{31}$$

i.e. eq. (7) is satisfied.

The nonzero components of H_{mnp} under the BPS condition are

$$\begin{aligned} H_{012} &= \frac{\eta_1}{4}(\sin\theta + \cos\theta\partial_2 X) \\ H_{01\alpha} &= \frac{\eta_1}{4}\cos\theta\partial_\alpha X \\ H_{\alpha\beta\gamma} &= \frac{\eta_2}{4}\epsilon_{\alpha\beta\gamma}\left(\partial_2 X + \frac{\sin\theta(1+(\partial X)^2)}{\cos\theta - \sin\theta\partial_2 X}\right) \\ H_{2\alpha\beta} &= -\frac{\eta_2}{4}\epsilon_{\alpha\beta\gamma}\partial^\gamma X\,. \end{aligned} \tag{32}$$

And

$$\begin{aligned} G^{\alpha\beta} &= 4\delta^{\alpha\beta}\frac{(\cos\theta - \sin\theta\partial_2 X)^2}{[-\eta_1\cos\theta + \sin\theta\partial_2 X + \eta_2\sqrt{1+(\partial X)^2}]^2} \\ G^{2\alpha} &= 4\partial_\alpha X\frac{\sin\theta(\cos\theta - \sin\theta\partial_2 X)}{[-\eta_1\cos\theta + \eta_1\sin\theta\partial_2 X + \eta_2\sqrt{1+(\partial X)^2}]^2} \\ G^{22} &= 4\frac{1 - \sin^2\theta\partial_\alpha X\partial^\alpha X}{[-\eta_1\cos\theta + \eta_1\sin\theta\partial_2 X + \eta_2\sqrt{1+(\partial X)^2}]^2}\,. \end{aligned} \tag{33}$$

The equation of motion for X is

$$G^{m'n'}\nabla_{m'}\partial_{n'}X = \frac{1}{1+(\partial X)^2}G^{m'n'}\partial_{m'}\partial_{n'}X = 0\,. \tag{34}$$

This equation is rewritten as follows.

$$\partial_\alpha\partial^\alpha X + \partial_2\partial^2 X + \partial_2\left(\frac{\sin\theta(1+(\partial X)^2)}{\cos\theta - \sin\theta\partial_2 X}\right) = 0\,. \tag{35}$$

Using eq. (34) or (35), the nontrivial components of the equation of motion for H_{mnp} are

$$\begin{aligned} G^{m'n'}\nabla_{m'}H_{n'01} &= G^{m'n'}\partial_{m'}H_{n'01} = 0 \\ G^{m'n'}\nabla_{m'}H_{n'p'q'} &= G^{m'n'}\Big(\partial_{m'}H_{n'p'q'} - \partial_{m'}\partial_{p'}X\partial_{r'}Xg^{r's'}H_{n's'q'} - \\ &\qquad - \partial_{m'}\partial_{q'}X\partial_{r'}Xg^{r's'}H_{n'p's'}\Big) = 0\,. \end{aligned} \tag{36}$$

It can be shown that these are satisfied by using eq. (34) or (35) further. Furthermore the Bianchi identity

$$\partial_{[m} H_{npq]} = 0 \tag{37}$$

is also satisfied. Hence all we should do is to solve eq. (34) or (35).

Solving eq. (35) seems very difficult. However, since we expect that the M2-brane is tilted, we can simplyfy this equation by using the rotated coordinate $(\bar{X}, \bar{z})$, as was done in [15]. $(\bar{X}, \bar{z})$ is related to $(X, z \equiv \sigma^2)$ as follows.

$$\begin{pmatrix} \bar{X} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ z \end{pmatrix} . \tag{38}$$

We denote the derivatives with respect to $\bar{z}$ and σ_α by $\bar{\partial}_z$ and $\bar{\partial}_\alpha$. Then

$$\begin{aligned} \partial_z \bar{z} &= \frac{1}{\cos\theta + \sin\theta \bar{\partial}_z \bar{X}} \\ \partial_\alpha \bar{z} &= -\frac{\sin\theta \bar{\partial}_\alpha \bar{X}}{\cos\theta + \sin\theta \bar{\partial}_z \bar{X}} . \end{aligned} \tag{39}$$

Eq. (35) is rewritten as follows.

$$\bar{\partial}_\alpha \bar{\partial}^\alpha \bar{X} + \bar{\partial}_z \bar{\partial}_z \bar{X} = 0 . \tag{40}$$

This can be easily solved. The solution with the boundary condition that the M5-brane worldvolume lies in the direction $X^{0,1,2,3,4,5}$ at the infinity is

$$\bar{X} = \sum_i \frac{Q_i}{(\bar{z}^2 + (\sigma^\alpha)^2) - (\bar{z}_i^2 + (\sigma_i^\alpha)^2)} + \tan\theta \bar{z} . \tag{41}$$

We can interpret this solution as tilted M2-branes emanating from $(\bar{z}_i, \sigma_i^\alpha)$. See figure 2. From this solution we can obtain the values of h_{abc} at the infinity of the worldvolume:

$$\begin{aligned} h_{012} &\to -\frac{1}{2} \frac{\eta_2 \sin\theta}{\cos\theta - \eta_1 \eta_2} \\ h_{345} &\to \frac{1}{2} \frac{\eta_2 \sin\theta}{\cos\theta - \eta_1 \eta_2} \\ H_{012} &\to \frac{1}{4} \eta_1 \sin\theta \\ H_{345} &\to \frac{1}{4} \eta_2 \tan\theta , \end{aligned} \tag{42}$$

with the other components vanishing. These values agrees with eqs. (12), (13), and (14). The coefficients Q_i are given in (45) below.

There is the critical value for H_{012} i.e. $H_{012} = \pm 1/4$ [2, 4]. At this value $\theta = \pm\pi/2$ and the M2-brane is parallel to the M5-brane.

Next we consider the dimensional reduction of this solution. If we compactify the direction m along the M5-brane, we have a D4 brane in type-IIA string theory and [5]

$$H_{mnp} = \frac{1}{4}(F_{np} + B_{np}) \,, \tag{43}$$

where F_{np} is the field strength of the gauge field on the D4-brane and B_{np} is (pullback of) the NS-NS 2-form. If we compactify the direction perpendicular to the M5-brane, we obtain a NS5-brane in a constant RR 3-form. Let us consider three cases. The first case is the double dimensional reduction along the direction X^1. We obtain the configuration that tilted F-strings ending on a D4-brane in constant B_{02}. The relationship between the constant NS-NS 2-form and the angle θ is

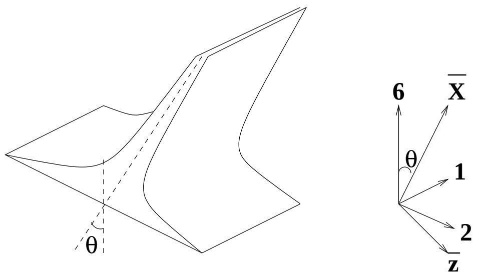

Figure 2: the solution on the M5-brane.

$$B_{02} = \eta_1 \sin\theta \,, \tag{44}$$

and the solution is in the same form as eq. (41). This result agrees with that of [16]. We can determine the coefficient Q_i from the argument in [17]:

$$Q_i = \pm\frac{4\pi^3\alpha^{3/2}g_s}{\Omega_3} = \pm\frac{4\pi^3\ell_p^3}{\Omega_3} \,, \tag{45}$$

where g_s is the string coupling constant, ℓ_p is the 11 dimensional Planck length and Ω_3 is the volume of S^3. The second case is the double dimensional reduction along the direction X^5. We obtain the configuration that tilted D2-branes ending on a D4-brane in constant B_{34}:

$$B_{34} = \eta_2 \tan\theta \,, \tag{46}$$

and the solution is in the same form as eq. (41). This result agrees with that of [15].

The third case is the dimensional reduction along the direction X^{10}. We obtain the configuration that tilted D2-branes ending on a NS5-brane in constant RR 3-form. The geometry of the solution is the same as in the 11 dimensional case.

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